$$\frac{d\lambda}{df} = -\frac{\lambda}{f}.$$

Substituting into (4) we obtain

$$\frac{\delta\lambda_g}{\lambda_g} = -\left(1+\frac{\lambda_g^2}{\lambda_c^2}\right)\frac{\delta f}{f}. \qquad (5)$$

Substituting again from (1a) and (5) into (1b) we obtain

$$\left\{1-\left(1+\frac{4\pi^2L^2}{(\theta+2\pi n)^2\lambda_c^2}\right)\frac{\delta f}{f}\right\} \cdot \frac{2\pi L(\theta+\delta\theta+2\pi n)}{\theta+2\pi n} = 2\pi L.$$

Or putting

$$\theta + 2\pi n = x; \qquad \frac{4\pi^2L^2}{\lambda_c^2} = \xi$$

we obtain

$$\frac{\delta f}{f}x^3 - x^2\delta\theta\left(1-\frac{\delta f}{f}\right) + x\xi\frac{\delta f}{f} + \delta\theta\xi\frac{\delta f}{f} = 0. \qquad (6)$$

In view of the fact that n is an integer, no error is introduced by the approximation involved in (4). The accuracy of the method depends solely on the accuracy to which θ and L have been measured. On the other hand, f must be measured to an accuracy just sufficient for an unambiguous determination of n from (6).

Accuracy in the measurement of θ is assured by a high-quality slotted line and variable-short-circuit and it is suggested that no less than 10 points be measured for each frequency to assure the accurate plotting of the iconocentre of the transformed unit circle.

The above described method has been successfully applied to the measurement of the propagation constant in a serrated waveguide filled with polystyrene.

EFRAIM WEISSBERG
Scientific Dept.
Ministry of Defence
Hakirya, Tel Aviv, Israel

An Automatic Microwave Dielectrometer*

The dielectric constant of low-loss dielectric materials can be measured accurately, rapidly, and automatically by proper utilization of an automatic microwave impedance instrument of the type which presents its output in Smith Chart form; *i.e.*, the magnitude and phase of the reflection coefficient. The method employed is essentially a modification of the slotted line technique which was originally described by Roberts and Von Hippel,[1] the difference being that the slotted line is replaced by an automatic impedance instrument with a direct reading scale.

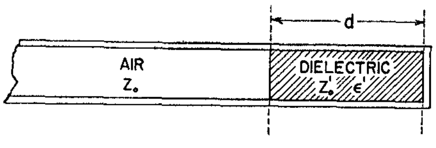

Fig. 1—Dielectric slug in short circuited section of waveguide.

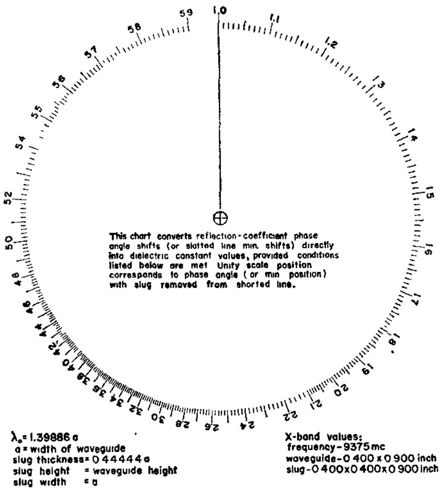

Fig. 2—Example of a direct reading dielectric constant scale.

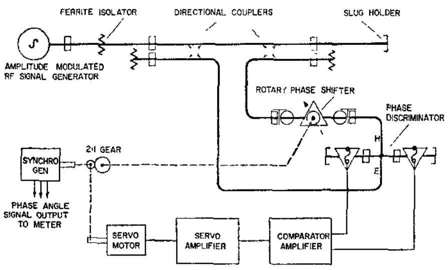

Fig. 3—Schematic diagram of an automatic dielectrometer.

A sample of the dielectric to be measured is accurately cut in the form of a slug which fills the waveguide in its transverse dimensions and is positioned against a reference short circuit, as shown in Fig. 1. From the derived relationship for this case available in the literature,[2] it will be found that the change in a slotted line minimum position, caused by insertion of the dielectric slug, can be related to the dielectric constant of the slug by

$$x = \frac{\lambda_g}{2\pi}\tan^{-1}\left[\frac{\lambda'_g}{\lambda_g}\tan\left(\frac{2\pi d}{\lambda'_g}\right)\right] - d$$

where

x = "min-shift" distance (minimum position without sample minus that with sample)
λ_g = wavelength in air-filled waveguide
λ'_g = wavelength in dielectric-filled waveguide
d = length of sample.

The particular expression for λ'_g will contain the dielectric constant, of course. Upon replacing the slotted line with an automatic impedance instrument which measures reflection coefficient, it becomes desirable to convert the "min-shift" distance, x, to the associated change in reflection coefficient phase angle, Δ_ϕ,

$$\Delta_\phi = \frac{4\pi x}{\lambda_g}.$$

One can then calculate values of Δ_ϕ for various values of dielectric constant, assuming fixed values for λ_g and d, and can construct circular scales which read directly in dielectric constant. An example of such a scale is shown in Fig. 2, calculated for 0.400 ×0.400×0.900 inch slugs in X-band waveguide at a frequency of 9375 mc. It is assumed that the relative permeability is unity and that λ'_g is given by the TE_{10} mode expression

$$\lambda'_g = \frac{\lambda_0}{\sqrt{\epsilon' - \left(\frac{\lambda_0}{2a}\right)^2}}$$

where

λ_0 = free space wavelength,
ϵ' = dielectric constant (relative)
a = width of waveguide.

Scales such as this convert reflection coefficient phase angle shifts directly into dielectric constant values, thereby permitting automatic indication. If variations in slug length or frequency are desired, a family of such scales can be constructed to cover the particular sets of conditions involved.

To use the scale, replace the Smith Chart with it on whatever output device the impedance instrument utilizes, usually an oscilloscope or a linear polar recorder. Next, align the unity scale position with the phase angle indicated for the reference short circuit by itself. Then simply insert the slugs against the short circuit and either record or read off the dielectric constant values.

The accuracy of this type of dielectrometer is dependent upon the accuracy with which the automatic impedance instrument can follow changes in reflection coefficient phase angle at a given frequency. Results comparable to those obtained from slotted-line measurements require a phase angle accuracy of about ±0.5 degrees. Instruments of this order of accuracy have been built,[3] utilizing a servo driven Fox type rotary phase shifter as the principal element in the phase measuring circuit.

If an automatic dielectrometer is desired, but an automatic impedance instrument does not happen to be available for such use, then the arrangement shown in Fig. 3 is suggested. It is based upon the reference impedance instrument.[3]

WILLIAM F. GABRIEL
Microwave Antennas and Components Branch
U. S. Naval Res. Lab.
Washington, D. C.

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[2] C. G. Montgomery, "Technique of Microwave Measurements," Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 11, Ch. 10, p. 627, (71); 1947.

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